

## Analysis of the $B \rightarrow \pi \ell^+ \ell^-$ decay in the standard model with fourth generation

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**ABSTRACT:** We investigate the influence of the fourth generation quarks on the branching ratio and the CP-asymmetry in  $B \rightarrow \pi \ell^+ \ell^-$  decay. Taking the  $|V_{t'd}V_{t'b}| \sim 0.001$  with phase about  $10^\circ$ , which is consistent with the  $\sin 2\phi_1$  of the CKM and the  $B_d$  mixing parameter  $\Delta m_{B_d}$ , we obtain that for both  $(\mu, \tau)$  channels the branching ratio, the magnitude of CP-asymmetry and lepton polarization depict strong dependency on the 4th generation quarks mass and mixing parameters. These results can serve as a good tool to search for new physics effects, precisely, to search for the fourth generation quarks ( $t', b'$ ) via its indirect manifestations in loop diagrams.

**KEYWORDS:** Beyond Standard Model, B-Physics.

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## 1. Introduction

The first evidence of new strong interactions may be a sufficiently massive fourth family observed at the LHC. The fourth family masses, of the leptons in particular, are constrained by the electroweak precision data, and this leads to signatures at the LHC that may imply early discovery. The discovery of a fourth family could potentially come quite early. The fourth family quarks and leptons are free to have mass mixing (CKM mixing) with the lighter fermions, and thus treelevel charged-current decays or some loop induced(FCNC) decays may be affected by the existence of the 4th generation top like quark( $t'$ ) (i.e, see [1]–[6]). We will discuss processes of this type that should be quite accessible at the LHC. There are constraints on a fourth family [7]. From the strong constraint on the number of light neutrinos, we know that the fourth family neutrino is heavy. The  $S$  and  $\rho$  parameters are sensitive to a fourth family, but the experimental limits on these parameters have been evolving over the years in such a way that the constraint on a fourth family has lowered. In addition, the masses of the fourth family leptons may be such as to produce negative  $S$  and  $T$ . As discussed in [8] and the reference therein the constraints from  $S$  and  $T$  do not prohibit the fourth family, but instead serve only to constrain the mass spectrum of the fourth family quarks and leptons. The implied masses for the fourth family leptons should make them particularly accessible at the LHC, with neutrino pair production providing the most interesting signatures. Any way, a sequential fourth family is theoretically attractive because it makes it possible that a theory of flavor is related to the breakdown of a simple family gauge symmetry [9]. In contrast, new fermions not having standard model quantum numbers would be more surprising and difficult to understand.

New Physics (NP) can be searched for in two ways: either by raising the available energy at colliders to produce new particles and reveal them directly, or by increasing the experimental precision on certain processes involving Standard Model (SM) particles as external states. The latter option, indirect search for NP, should be pursued using processes that are forbidden, very rare or precisely calculable in the SM. In this respect, Flavor Changing Neutral Current (FCNC) and CP-violating processes are among the most

powerful probes of NP, since in the SM they cannot arise at the tree-level and even at the loop level they are strongly suppressed by the GIM mechanism. Furthermore, in the quark sector they are all calculable in terms of the CKM matrix, and in particular of the parameters  $\bar{\rho}$  and  $\bar{\eta}$  in the generalized Wolfenstein parametrization [10]. Unfortunately, in many cases a deep understanding of hadronic dynamics is required in order to be able to extract the relevant short-distance information from measured processes. Lattice QCD and QCD sum rules allow us to compute the necessary hadronic parameters in many processes. Indeed, the Unitarity Triangle Analysis (UTA) with Lattice QCD input is extremely successful in determining  $\bar{\rho}$  and  $\bar{\eta}$  and in constraining NP contributions [11–15].

Once the CKM matrix is precisely determined by means of the UTA, it is possible to search for NP contributions. FCNC and CP-violating are indeed the most sensitive probes of NP contributions to penguin operators. Rare decays, induced by flavor changing neutral current (FCNC)  $b \rightarrow s(d)$  transitions is at the forefront of our quest to understand flavor and the origins of CPV, offering one of the best probes for New Physics (NP) beyond the Standard Model (SM) [16]–[18]. In addition, there are important QCD corrections, which have recently been calculated in the NNLL [19]. Moreover,  $b \rightarrow s(d)\ell^+\ell^-$  decay is also very sensitive to the new physics beyond SM. New physics effects manifest themselves in rare decays in two different ways, either through new combinations to the Wilson coefficients or through the new structure of the operator in the effective Hamiltonian, which is absent in the SM. A crucial problem in the new physics search within flavor physics is the optimal separation of new physics effects from uncertainties. It is well known that inclusive decay modes are dominated partonic contributions; non-perturbative corrections are in general rather small [20]. Also, ratios of exclusive decay modes such as asymmetries for  $B \rightarrow K(K^*, \rho, \gamma)\ell^+\ell^-$  decay [21]–[30] are well studied for new-physics search. Here large parts of the hadronic uncertainties, partially, cancel out.

In this paper, we investigate the possibility of searching for new physics in the  $B \rightarrow \pi\ell^+\ell^-$  decay using the SM with four generations of quarks( $b', t'$ ). The fourth quark ( $t'$ ), like  $u, c, t$  quarks, contributes in the  $b \rightarrow s(d)$  transition at loop level. It would clearly change the branching ratio and CP-asymmetry. Note that, fourth generation effects on the branching ratio have been widely studied in baryonic and semileptonic  $b \rightarrow s$  transition [31]–[40]. But, there isn't any study related to the  $b \rightarrow d$  transitions .

The sensitivity of the physical observable to the existence of fourth generation quarks in the  $B \rightarrow K^*\ell^+\ell^-$  decay is investigated in [6] and it is obtained that the CP asymmetry is very sensitive to the fourth generation parameters ( $m_{t'}, V_{t'b}V_{t'd}^*$ ). In this connection, it is natural to ask whether the branching ratio, CP-asymmetry and lepton polarization in  $B \rightarrow \pi\ell^+\ell^-$  are sensitive to the fourth generation parameters in the same way. In the present work, we try to answer to these questions.

The paper is organized as follows: In section 2, using the effective hamiltonian, the general expressions for the matrix element and CP asymmetry of  $B \rightarrow \pi\ell^+\ell^-$  decay is derived. Section 3 is devoted to calculations of lepton polarization. In section 4, we investigate the sensitivity of the above mentioned physical observable to the fourth generation parameters ( $m_{t'}, V_{t'b}V_{t'd}^*$ ).

## 2. Matrix element, differential decay rate and CP asymmetry

With a sequential fourth generation, the Wilson coefficients  $C_7$ ,  $C_9$  and  $C_{10}$  receive contributions from the  $t'$  quark loop, which we will denote as  $C_{7,9,10}^{\text{new}}$ . Because a sequential fourth generation couples in a similar way to the photon and W, the effective Hamiltonian relevant for  $b \rightarrow d\ell^+\ell^-$  decay has the following formula:

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{td}^* \sum_{i=1}^{10} C_i(\mu) \mathcal{O}_i(\mu), \quad (2.1)$$

where the full set of the operators  $\mathcal{O}_i(\mu)$  and the corresponding expressions for the Wilson coefficients  $C_i(\mu)$  in the SM are given in [41]–[43]. As it has already been noted, the fourth generation up type quark  $t'$  is introduced in the same way as  $u$ ,  $c$ ,  $t$  quarks introduce in the SM, so, new operators do not appear and clearly the full operator set is exactly the same as in SM. The fourth generation changes the values of the Wilson coefficients  $C_7(\mu)$ ,  $C_9(\mu)$  and  $C_{10}(\mu)$ , via virtual exchange of the fourth generation up type quark  $t'$ . The above mentioned Wilson coefficients will explicitly change as

$$\lambda_t C_i \rightarrow \lambda_t C_i^{\text{SM}} + \lambda_{t'} C_i^{\text{new}}, \quad (2.2)$$

where  $\lambda_f = V_{fb}^* V_{fd}$ . The unitarity of the  $4 \times 4$  CKM matrix leads to

$$\lambda_u + \lambda_c + \lambda_t + \lambda_{t'} = 0. \quad (2.3)$$

It follows that

$$\lambda_t C_i^{\text{SM}} + \lambda_{t'} C_i^{\text{new}} = \lambda_c C_i^{\text{SM}} + \lambda_{t'} (C_i^{\text{new}} - C_i^{\text{SM}}) \quad (2.4)$$

It is clear that, for the  $m_{t'} \rightarrow m_t$  or  $\lambda_{t'} \rightarrow 0$ ,  $\lambda_{t'} (C_i^{\text{new}} - C_i^{\text{SM}})$  term vanishes, as required by the GIM mechanism. One can also write  $C_i$ 's in the following form

$$\begin{aligned} C_7^{\text{tot}}(\mu) &= C_7^{\text{SM}}(\mu) + \frac{\lambda_{t'}}{\lambda_t} C_7^{\text{new}}(\mu), \\ C_9^{\text{tot}}(\mu) &= C_9^{\text{SM}}(\mu) + \frac{\lambda_{t'}}{\lambda_t} C_9^{\text{new}}(\mu), \\ C_{10}^{\text{tot}}(\mu) &= C_{10}^{\text{SM}}(\mu) + \frac{\lambda_{t'}}{\lambda_t} C_{10}^{\text{new}}(\mu), \end{aligned} \quad (2.5)$$

where the last terms in these expressions describe the contributions of the  $t'$  quark to the Wilson coefficients.  $\lambda_{t'}$  can be parameterized as:

$$\lambda_{t'} = V_{t'b}^* V_{t'd} = r_{db} e^{i\phi_{db}} \quad (2.6)$$

In deriving eq. (2.5), we factored out the term  $V_{tb}^* V_{td}$  in the effective Hamiltonian given in eq. (2.1). The explicit forms of the  $C_i^{\text{new}}$  can be easily obtained from the corresponding expression of the Wilson coefficients in SM by substituting  $m_t \rightarrow m_{t'}$  (see [41, 42]). If the  $d$

$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$	$C_7^{\text{SM}}$	$C_9^{\text{SM}}$	$C_{10}^{\text{SM}}$
-0.248	1.107	0.011	-0.026	0.007	-0.031	-0.313	4.344	-4.669

**Table 1:** The numerical values of the Wilson coefficients at  $\mu = m_b$  scale within the SM. The corresponding numerical value of  $C^0$  is 0.362.

quark mass is neglected, the above effective Hamiltonian leads to following matrix element for the  $b \rightarrow d\ell^+\ell^-$  decay

$$\mathcal{H}_{\text{eff}} = \frac{G_F\alpha}{2\sqrt{2}\pi} V_{tb}V_{td}^* \left[ C_9^{\text{tot}} \bar{d}\gamma_\mu(1-\gamma_5)b\bar{\ell}\gamma_\mu\ell + C_{10}^{\text{tot}} \bar{d}\gamma_\mu(1-\gamma_5)b\bar{\ell}\gamma_\mu\gamma_5\ell - 2C_7^{\text{tot}} \frac{m_b}{q^2} \bar{d}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b\bar{\ell}\gamma_\mu\ell \right], \quad (2.7)$$

where  $q^2 = (p_1 + p_2)^2$  and  $p_1$  and  $p_2$  are the final leptons four-momenta. The effective coefficient  $C_9^{\text{eff}}$  can be written in the following form:

$$C_9^{\text{eff}} = \xi_1 + \frac{\lambda_u}{\lambda_t} \xi_2 + Y(s'), \quad (2.8)$$

where  $s' = q^2/m_b^2$  and the function  $Y(s')$  denotes the perturbative part coming from one loop matrix elements of four quark operators [41, 43]. The explicit expressions for  $\xi_1$ ,  $\xi_2$ , and the values of  $C_i$  in the SM can be found in [41, 43].

In addition to the short distance contribution,  $Y_{\text{per}}(s')$  receives also long distance contributions, which have their origin in the real  $c\bar{c}$  and  $u\bar{u}$  intermediate states. The resonances are introduced by the Breit-Wigner distribution through the replacement [44]–[46]

$$Y(s') = Y_{\text{per}}(s') + \frac{3\pi}{\alpha^2} C^{(0)} \sum_{V_i=\psi_i} \kappa_i \frac{m_{V_i}\Gamma(V_i \rightarrow \ell^+\ell^-)}{m_{V_i}^2 - s'm_b^2 - im_{V_i}\Gamma_{V_i}}, \quad (2.9)$$

where  $C^{(0)} = 3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6$ . The phenomenological parameters  $\kappa_i$  can be fixed from experimental measurements of semileptonic B decays (i.e,  $\mathcal{B}(B \rightarrow K^*V_i \rightarrow K^*\ell^+\ell^-) = \mathcal{B}(B \rightarrow K^*V_i)\mathcal{B}(V_i \rightarrow \ell^+\ell^-)$ , where the data for the right hand side is given in [47]. For the lowest resonances  $J/\psi$  and  $\psi'$  one can use  $\kappa = 1.65$  and  $\kappa = 2.36$ , respectively (see [48]).

One has to sandwich the inclusive effective hamiltonian between initial hadron state  $B(p_B)$  and final hadron state  $\pi(p_\pi)$  to obtain the matrix element for the exclusive decay  $B(p_B) \rightarrow \pi(p_\pi)\ell^+(p_+)\ell^-(p_-)$ . It follows from eq. (2.7) that in order to calculate the decay width and other physical observable of the exclusive  $B \rightarrow \pi\ell^+\ell^-$  decay, the following matrix elements in terms of form factors

$$\langle \pi(p_\pi) | \bar{d}\gamma_\mu(1-\gamma^5)b | B(p_B) \rangle = f^+(q^2)(p_\pi + p_B)_\mu + f^-(q^2)q_\mu, \quad (2.10)$$

$$\langle \pi(p_\pi) | \bar{d}i\sigma_{\mu\nu}q^\nu(1+\gamma^5)b | B(p_B) \rangle = [q^2(p_\pi + p_B)_\mu - q_\mu(m_B^2 - m_\pi^2)]f_\nu(q^2), \quad (2.11)$$

have to be calculated. In other words, the exclusive  $B \rightarrow \pi\ell^+\ell^-$  decay which is described in terms of the matrix elements of the quark operators given in eq. (2.7) over meson

states, can be parameterized in terms of form factors ( $f^+$ ,  $f^-$  and  $f_v$ ). We observe that in calculating the physical observable at hadronic level, we face the problem of computing the form factors. This problem is related to the nonperturbative sector of QCD and it can be solved only in framework a nonperturbative approach. In the present work, we will use of the results the constituent quark model predictions for the form factors.

Now, we can obtain the matrix element which is as follows:

$$M^{B \rightarrow \pi} = \frac{G_F \alpha}{2\sqrt{2}\pi} V_{tb} V_{td}^* \left\{ (2Ap_\pi^\mu + Bq^\mu) \bar{\ell} \gamma_\mu \ell + (2Gp_\pi^\mu + Dq^\mu) \bar{\ell} \gamma_\mu \gamma^5 \ell \right\}, \quad (2.12)$$

where

$$A = C_9^{\text{new}} f^+ - 2m_B C_7^{\text{new}} f_v, \quad (2.13)$$

$$B = C_9^{\text{new}} (f^+ + f^-) + 2 \frac{m_B}{q^2} C_7^{\text{new}} f_v (m_B^2 - m_\pi^2 - q^2),$$

$$G = C_{10}^{\text{new}} f^+,$$

$$D = C_{10}^{\text{new}} (f^+ + f^-),$$

From this expression of the matrix element, for the unpolarized differential decay width we get the following result:

$$\left( \frac{d\Gamma^\pi}{ds} \right)_0 = \frac{G_F^2 \alpha^2}{2^{10} \pi^5} |V_{tb} V_{td}^*|^2 m_B^3 v \sqrt{\lambda_\pi} \Delta_\pi, \quad (2.14)$$

$$\begin{aligned} \Delta_\pi = & \frac{1}{3} m_B^2 \lambda_\pi (3 - v^2) (|A|^2 + |G|^2) + 16 m_\ell^2 r_\pi |G|^2 + 4 m_\ell^2 s |D|^2 \\ & + 8 m_\ell^2 (1 - r_\pi - s) \text{Re}[GD^*], \end{aligned} \quad (2.15)$$

with  $r_\pi = m_\pi^2/m_B^2$ ,  $\lambda_\pi = r_\pi^2 + (s-1)^2 - 2r_\pi(s+1)$ ,  $v = \sqrt{1 - \frac{4\ell^2}{s}}$  and  $t = m_\ell/m_B$ .

Another physical quantity is normalized CP violating asymmetry which can be defined for both polarized and unpolarized leptons. We aim to obtain normalized CP violating asymmetry for the unpolarized leptons. The standard definition are given as:

$$A_{CP}^\pi(\hat{s}) = \frac{\left( \frac{d\Gamma^\pi}{d\hat{s}} \right)_0 - \left( \frac{d\bar{\Gamma}^\pi}{d\hat{s}} \right)_0}{\left( \frac{d\Gamma^\pi}{d\hat{s}} \right)_0 + \left( \frac{d\bar{\Gamma}^\pi}{d\hat{s}} \right)_0} = \frac{\Delta_\pi - \bar{\Delta}_\pi}{\Delta_\pi + \bar{\Delta}_\pi}, \quad (2.16)$$

where

$$\frac{d\Gamma^\pi}{d\hat{s}} = \frac{d\Gamma^\pi(b \rightarrow d\ell^+\ell^-)}{d\hat{s}}, \quad \text{and}, \quad \frac{d\bar{\Gamma}^\pi}{d\hat{s}} = \frac{d\bar{\Gamma}^\pi(b \rightarrow d\ell^+\ell^-)}{d\hat{s}},$$

and  $(d\bar{\Gamma}^\pi/d\hat{s})_0$  can be obtained from  $(d\Gamma^\pi/d\hat{s})_0$  by making the replacement

$$C_9^{\text{eff}} = \xi_1 + \lambda_u \xi_2 \rightarrow \bar{C}_9^{\text{eff}} = \xi_1 + \lambda_u^* \xi_2. \quad (2.17)$$

Using this definition and the expression for  $\Delta_\pi(\hat{s})$  the CP violating asymmetry contributed from SM3 and new contribution from SM4 are:

$$A_{CP}^\pi(\hat{s}) = \frac{-\Sigma^{\text{SM}} - \Sigma^{\text{new}}}{\Delta_\pi^1 + \Sigma^{\text{SM}} + \Sigma^{\text{new}}} \quad (2.18)$$

where

$$\Sigma^{\text{SM}}(\hat{s}) = 4\text{Im}(\lambda_u) \left\{ f^{+2} \text{Im}(\xi_1^* \xi_2) + 2f^+ f_v m_B \text{Im}(C_7 \xi_2^*) \right\}, \quad (2.19)$$

$$\begin{aligned} \Sigma^{\text{new}}(\hat{s}) = & 4\text{Im}\left(\frac{\lambda_{t'}}{\lambda_t}\right) \left\{ 2f^+ f_v m_B [\text{Im}(c_7 c_9^{\text{new}*}) - \text{Im}(c_7^{\text{new}} \xi_1^*)] + f^{+2} \text{Im}(c_9^{\text{new}} \xi_1^*) \right\} \quad (2.20) \\ & + 4\text{Im}\left(\frac{\lambda_{t'}}{\lambda_t} \lambda_u\right) \left\{ 2f^+ f_v m_B \text{Im}(c_7 \xi_2^*) + f^{+2} \text{Im}(\xi_1^* \xi_2) \right\} \\ & + 4\text{Im}\left(\frac{\lambda_{t'}}{\lambda_t^*} \lambda_u\right) \left\{ -f^{+2} \text{Im}(c_9^{\text{new}} \xi_2^*) + 2f^+ f_v m_B \text{Im}(c_7^{\text{new}} \xi_2^*) \right\} \\ & + 4\text{Im}(\lambda_u) \left| \frac{\lambda_{t'}}{\lambda_t} \right|^2 \left\{ f^{+2} \text{Im}(c_9^{\text{new}} \xi_2) - 2f^+ f_v m_B \text{Im}(c_7^{\text{new}} \xi_2^*) \right\}, \end{aligned}$$

and

$$\Delta_\pi^1 = \frac{3\Delta_\pi}{m_B^2 \lambda_\pi (3 - v^2)}. \quad (2.21)$$

From this expression, it is firstly easy to see that in the  $\lambda_{t'} \rightarrow 0$  the SM3 result can be obtained. Secondly, when  $m_{t'} \rightarrow m_t$  the result of the SM4 coincide with the SM3 as it has to be seen (see figures), even if it is not obvious from the expressions.

### 3. Lepton polarization

In order to now calculate the polarization asymmetries of the lepton defined in the effective four fermion interaction of eq. (2.12), we must first define the orthogonal vectors  $S$  in the rest frame of  $\ell^-$  (where its vector is the polarization vector of the lepton). Note that, we use the subscripts  $L$ ,  $N$  and  $T$  to correspond to the leptons being polarized along the longitudinal, normal and transverse directions, respectively.

$$\begin{aligned} S_L^\mu &\equiv (0, \mathbf{e}_L) = \left( 0, \frac{\mathbf{p}_-}{|\mathbf{p}_-|} \right), \\ S_N^\mu &\equiv (0, \mathbf{e}_N) = \left( 0, \frac{\mathbf{p}_\pi \times \mathbf{p}_-}{|\mathbf{p}_\pi \times \mathbf{p}_-|} \right), \\ S_T^\mu &\equiv (0, \mathbf{e}_T) = (0, \mathbf{e}_N \times \mathbf{e}_L), \end{aligned} \quad (3.1)$$

where  $\mathbf{p}_-$  and  $\mathbf{p}_\pi$  are the three momenta of the  $\ell^-$  and  $\pi$  particles, respectively. The longitudinal unit vectors is boosted to the CM frame of  $\ell^- \ell^+$  by Lorentz transformation:

$$S_L^\mu = \left( \frac{|\mathbf{p}_-|}{m_\ell}, \frac{E_\ell \mathbf{p}_-}{m_\ell |\mathbf{p}_-|} \right),$$

while the other two vectors remain unchanged. The polarization asymmetries can now be calculated using the spin projector  $\frac{1}{2}(1 + \gamma_5 \mathcal{S})$  for  $\ell^-$ .

Provided the above expressions, we now define the single lepton polarization. The definition of the polarized and normalized differential decay rate is:

$$\frac{d\Gamma^\pi(s, \vec{n})}{ds} = \frac{1}{2} \left( \frac{d\Gamma^\pi}{ds} \right)_0 [1 + P_i^\pi \vec{e}_i \cdot \vec{n}], \quad (3.2)$$

	$f(0)$	$\sigma_1$	$\sigma_2$
$f_+$	0.29	0.48	
$F_0$	0.29	0.76	0.28
$f_v$	0.28	0.48	

**Table 2:**  $B \rightarrow \pi$  transition form factors in the constituent quark model.

where a sum over  $i = L, T, N$  is implied. Polarized components  $P_i^\pi$  in eq. (3.2) are as follows:

$$P_i^\pi = \frac{d\Gamma^\pi(\vec{n} = \vec{e}_i)d\hat{s} - d\Gamma^\pi(\vec{n} = -\vec{e}_i)/d\hat{s}}{d\Gamma^\pi(\vec{n} = \vec{e}_i)/d\hat{s} + d\Gamma^\pi(\vec{n} = -\vec{e}_i)/d\hat{s}}, \quad (3.3)$$

As a result, the different components of the  $P_i^\pi$  are given:

$$P_L^\pi = \frac{4m_B^2}{3\Delta_\pi} v \lambda_\pi \text{Re}[AG^*], \quad (3.4)$$

$$P_T^\pi = \frac{m_B^2}{\sqrt{\hat{s}}\Delta_\pi} \pi \sqrt{\lambda_\pi t} \left( \text{Re}[AD^*]\hat{s} + \text{Re}[AG^*](1 - r_\pi - \hat{s}) \right), \quad (3.5)$$

$$P_N^\pi = 0.$$

A few words here are in order. Firstly,  $P_N^\pi$  is zero in SM3 and SM4. It might be gained non-zero value in the case that the type of the interaction change (i.e. scalar or tensor type interactions may contribute). Secondly,  $P_T^\pi$  is proportional to the lepton mass and it is negligible for electron case in SM3, considering SM4, it will be measurable in the case that Wilson coefficients enhanced significantly by  $m_{t'}$ .

#### 4. Numerical analysis

In this section, we will study the dependence of the total branching ratio, CP asymmetry and lepton polarizations as well as combined lepton polarization to the fourth quark mass ( $m_{t'}$ ) and the product of quark mixing matrix elements ( $V_{t'b}^* V_{t'd} = r_{db} e^{i\phi_{ab}}$ ). The main input parameters in the calculations are the form factors. we have used the results of the constituent quark model [50], where the form factors  $f_T$  and  $f_+$  can be parameterized as:

$$f(q^2) = \frac{f(0)}{(1 - q^2/T_f^2)[1 - \sigma_1 q^2/M^2 + \sigma_2 q^4/M^4]}. \quad (4.1)$$

In this model,  $f_-$  is defined slightly different and it is as:

$$f(q^2) = \frac{f(0)}{[1 - \sigma_1 q^2/M^2 + \sigma_2 q^4/M^4]}. \quad (4.2)$$

The parameters  $f(0)$ ,  $\sigma_i$ 's can be found in table 2.

The other input parameters used in our numerical analysis are as follows:

$$\begin{aligned} m_B &= 5.28 \text{ GeV}, m_b = 4.8 \text{ GeV}, m_c = 1.5 \text{ GeV}, m_\tau = 1.77 \text{ GeV}, m_e = 0.511 \text{ MeV}, \\ m_\mu &= 0.105 \text{ GeV}, m_\rho = 0.77 \text{ GeV}, m_d = m_u = m_\pi = 0.14 \text{ GeV}, \\ |V_{cb}| &= 0.044, \alpha^{-1} = 129, G_f = 1.166 \times 10^{-5} \text{ GeV}^{-2}, \tau_B = 1.56 \times 10^{-12} \text{ s}. \end{aligned} \quad (4.3)$$



In the Wolfenstein parametrization of the CKM matrix [10],  $\lambda_u$  is written as:

$$\lambda_u = \frac{\rho(1 - \rho) - \eta^2 - i\eta}{(1 - \rho)^2 + \eta^2} + O(\lambda^2). \tag{4.4}$$

Furthermore, we use the relation

$$\frac{|V_{tb}V_{td}^*|^2}{|V_{cb}|^2} = \lambda^2[(1 - \rho)^2 + \eta^2] + \mathcal{O}(\lambda^4) \tag{4.5}$$

where  $\lambda = \sin \theta_C \simeq 0.221$  and adopt the values of the Wolfenstein parameters as  $\rho = 0.25$  and  $\eta = 0.34$ .

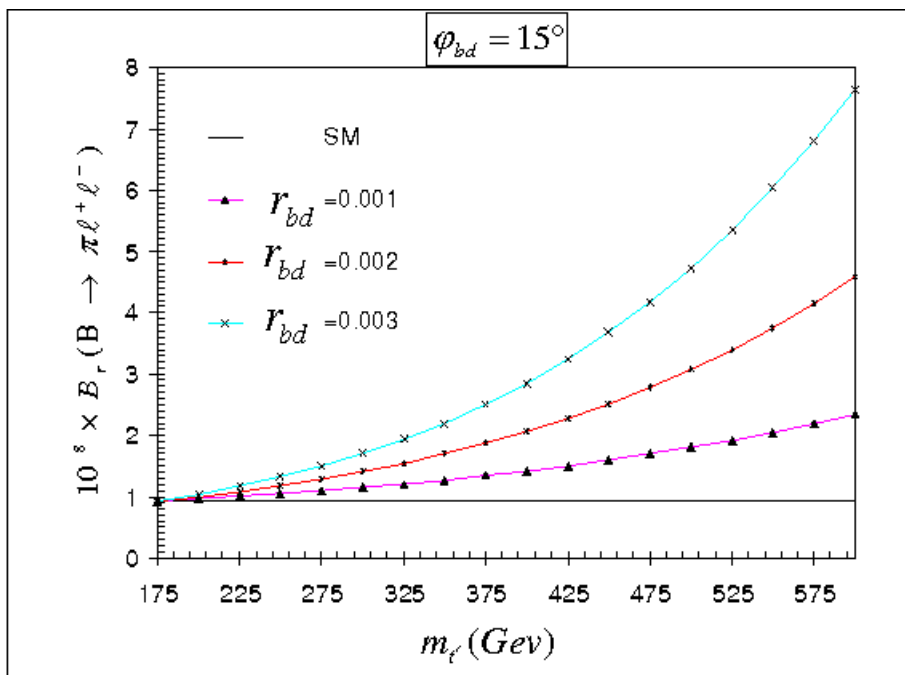
In order to perform quantitative analysis of the total branching ratio, CP asymmetry and the lepton polarizations, the values of the new parameters( $m_{t'}$ ,  $r_{db}$ ,  $\phi_{db}$ ) are needed. In the foregoing numerical analysis, we alter  $m_{t'}$  in the range  $175 \leq m_{t'} \leq 600\text{GeV}$ . The lower range is because of the fact that the fourth generation up quark should be heavier than the third ones( $m_t \leq m_{t'}$ ) [7]. The upper range comes from the experimental bounds on the  $\rho$  and  $S$  parameters of SM, furthermore, a mass greater than the 600GeV will also contradict with partial wave unitarity [7]. As for mixing, we use the result of Ref [51] where it is obtained that  $|V_{t'd}V_{t'b}| \sim 0.001$  with phase about  $10^\circ$  is consistent with the  $\sin 2\phi_1$  of the CKM and the  $B_d$  mixing parameter  $\Delta m_{B_d}$  [51].

Still, one more step can be proceeded. From explicit expressions of the physical observable one can easily see that they depend on both  $\hat{s}$  and the new parameters( $m_{t'}$ ,  $r_{db}$ ). One may eliminate the dependence of these quantities on one of the variables. We eliminate the variable  $\hat{s}$  by performing integration over  $\hat{s}$  in the allowed kinematical region. The total branching ratio and the averaged lepton polarizations are defined as

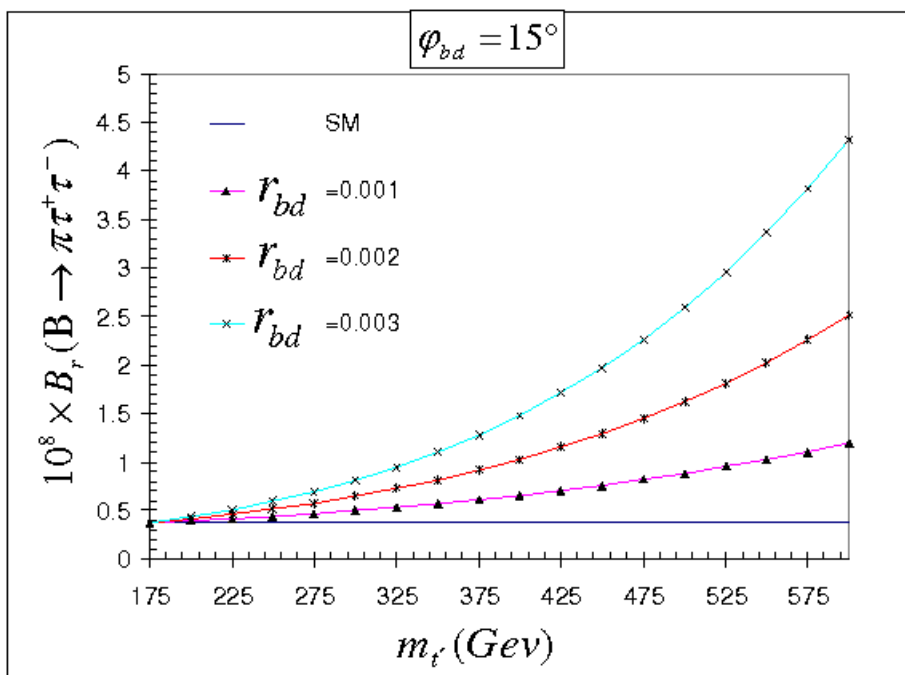
$$\begin{aligned} \mathcal{B}_r &= \int_{4m_\ell^2/m_B^2}^{(1-\sqrt{\hat{r}_\pi})^2} \frac{d\mathcal{B}}{d\hat{s}} d\hat{s}, \\ \langle P_i^\pi(A_{CP}^\pi) \rangle &= \frac{\int_{4m_\ell^2/m_B^2}^{(1-\sqrt{\hat{r}_\pi})^2} P_i^\pi(A_{CP}^\pi) \frac{d\mathcal{B}}{d\hat{s}} d\hat{s}}{\mathcal{B}_r}. \end{aligned} \tag{4.6}$$

Figures 1–8 depict the dependence of the total branching ratio, unpolarized averaged CP asymmetry and averaged lepton polarization for various  $r_{db}$  in terms of  $m_{t'}$ . We should note, here, that the dependency for various  $\phi_{db} \sim \{0^\circ - 30^\circ\}$  is too weak, then we show the results just for  $\phi_{db} = 15^\circ$ . Looking at these figures, the following outcomes are in order.

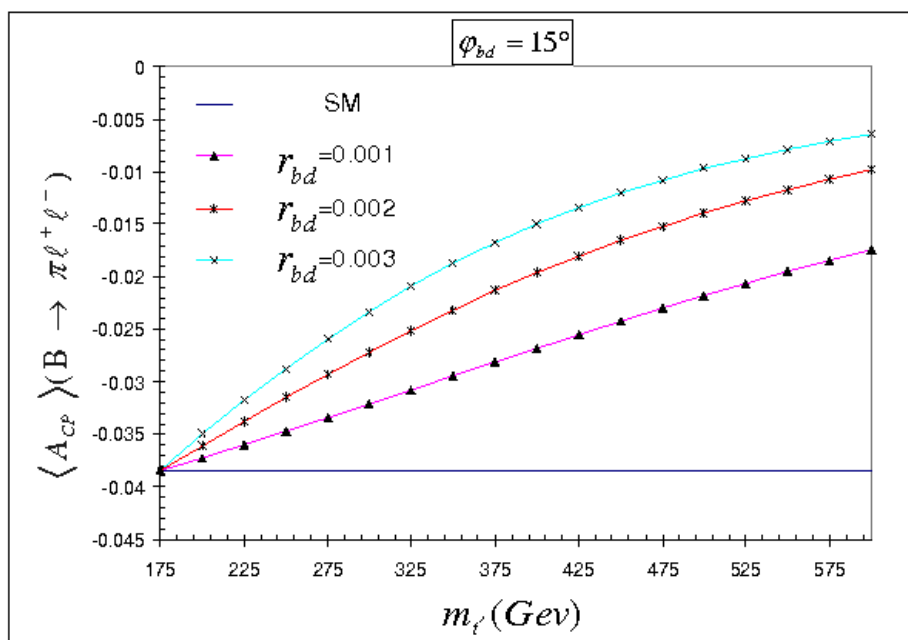
- $\mathcal{B}_r$  strongly depends on the fourth quark mass ( $m_{t'}$ ) and the product of quark mixing matrix elements( $r_{db}$ ) for both  $\mu$  and  $\tau$  channels. Furthermore, for both channels,  $\mathcal{B}_r$  is an increasing function of both  $m_{t'}$  and  $r_{db}$ .
- $P_L^\pi$  and  $A_{CP}^\pi$  are independent of the lepton mass (see eq. (3.4) and (2.16)) as a result, for given values of  $\hat{s}$  they are the same for  $e$ ,  $\mu$ , and  $\tau$  channels. The situation is different for the  $\langle P_L^\pi \rangle$  and  $\langle A_{CP}^\pi \rangle$ , those values for  $\tau$  channel are less than as for  $\mu$  and  $e$  channel, because the phase integral depends on the lepton mass ( $m_\ell$ ) (see



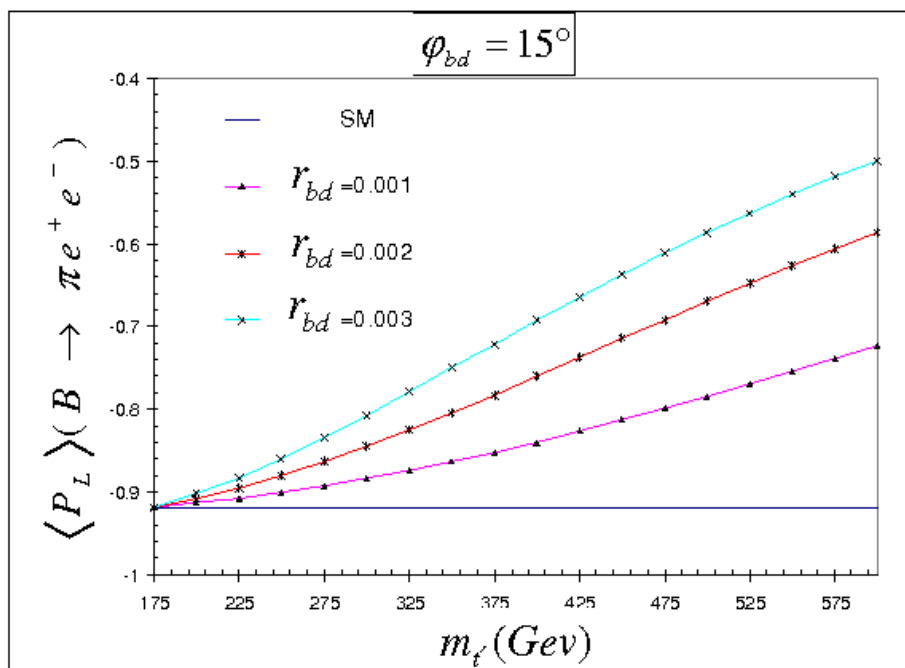
**Figure 1:** The dependence of the branching ratio of  $B \rightarrow \pi \ell^- \ell^+$  where,  $\ell = e, \mu$ , on  $m_{t'}$  for  $r_{db} = 0.001, 0.002, 0.003$ .



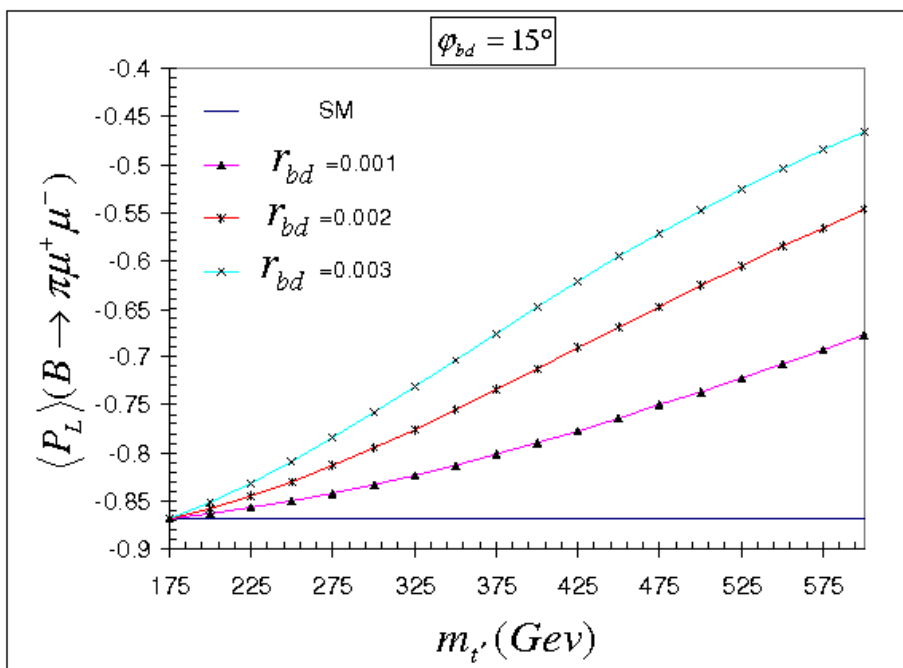
**Figure 2:** The same as in figure 1, but for the  $\tau$  lepton.



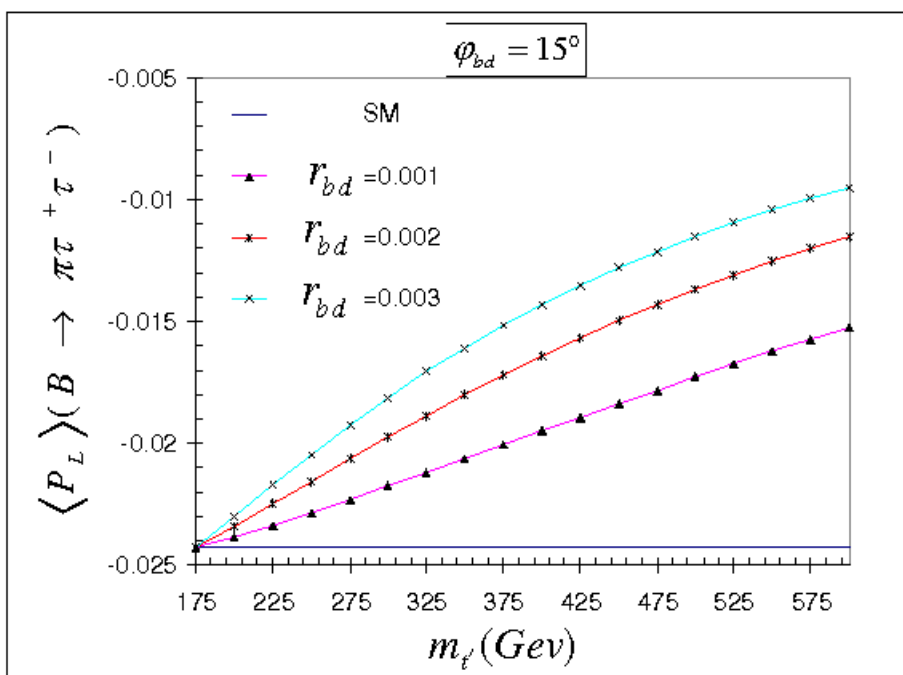
**Figure 3:** The dependence of the  $\langle A_{CP} \rangle$  on  $m_{\nu'}$  for  $r_{db} = 0.001, 0.002, 0.003$ , where  $\ell = e, \mu$ .



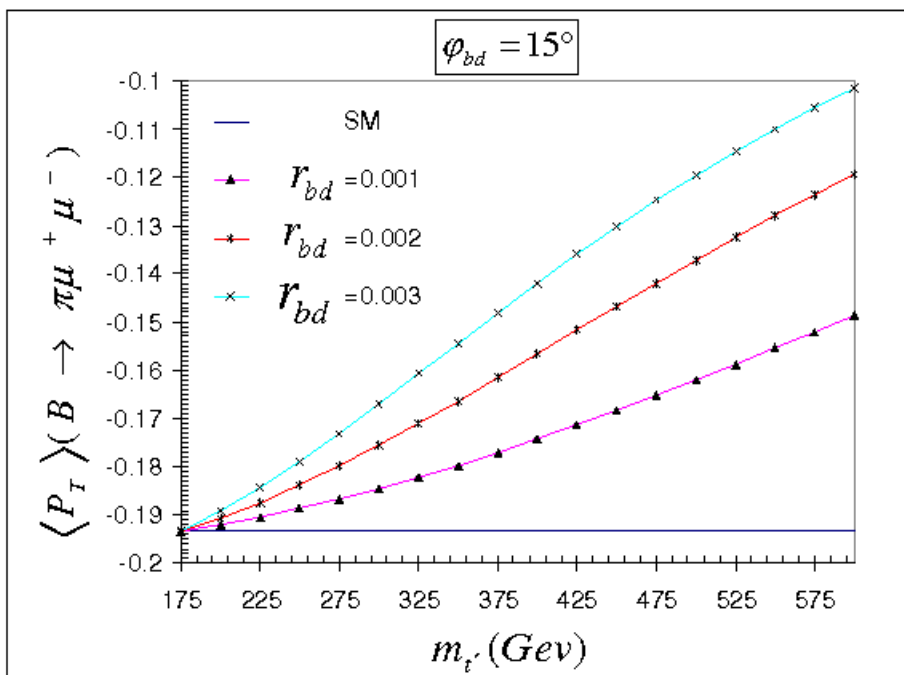
**Figure 4:** The dependence of the  $\langle P_L \rangle$  for  $e$  lepton, on  $m_{\nu'}$  for  $r_{db} = 0.001, 0.002, 0.003$ .



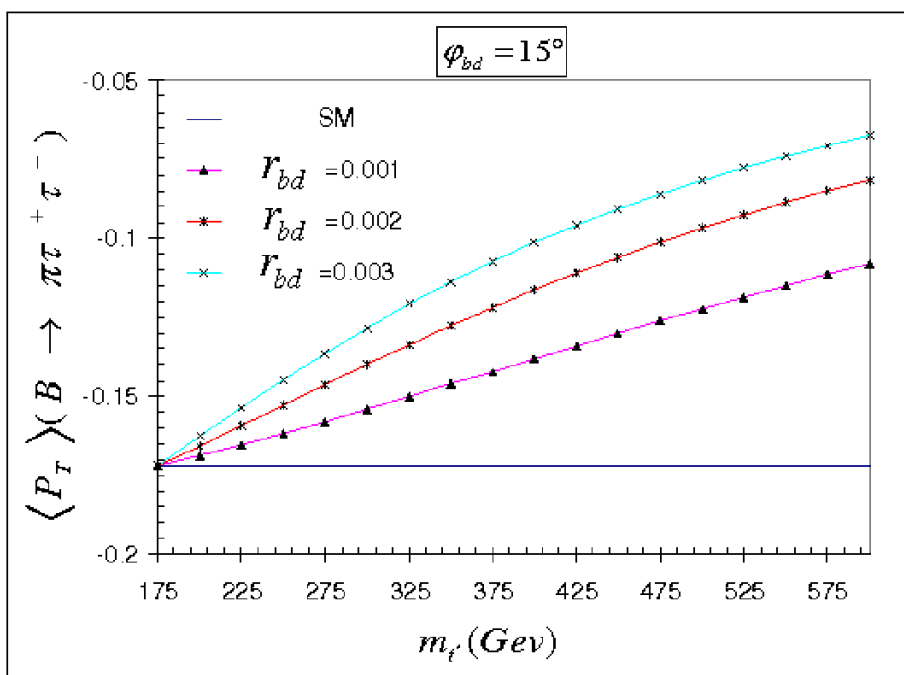
**Figure 5:** The same as in figure 4, but for the  $\mu$  lepton.



**Figure 6:** The same as in figure 4, but for the  $\tau$  lepton.



**Figure 7:** The dependence of the  $\langle P_T \rangle$  for  $\mu$  lepton, on  $m_{\nu'}$  for  $r_{db} = 0.001, 0.002, 0.003$ .



**Figure 8:** The same as in figure 7, but for the  $\tau$  lepton.

eq. (4.6)). The SM3 value of  $\langle P_L^\pi \rangle$  and  $\langle A_{CP}^\pi \rangle$  are negligible for the  $\tau$  channel ( $\sim 2\%$  and  $\sim 0.1\%$ , respectively). The SM4 suppress those approximately to zero. On the other hand,  $\langle P_L^\pi \rangle$  and  $\langle A_{CP}^\pi \rangle$  for  $e, \mu$  channels are strongly depends to the SM4 parameters. Moreover, their magnitudes are a decreasing function of the  $r_{db}$  and  $m_{t'}$ .

- Although,  $\langle P_T^\pi \rangle$  strongly depends on the fourth quark mass ( $m_{t'}$ ) and the product of quark mixing matrix elements ( $r_{sb}$ ) for both  $\mu$  and  $\tau$  channels. But, its magnitude is a decreasing function of both  $m_{t'}$  and  $r_{sb}$ . So, the existence of fourth generation of quarks will suppress the magnitude of  $\langle P_T^\pi \rangle$ .

In conclusion, we presented the systematic analysis of the  $B \rightarrow \pi \ell^- \ell^+$  decay, by using the SM with fourth generation of quarks. The sensitivity of the total branching ratio, CP asymmetry and lepton polarization on the new parameters, coming out of fourth generations, was studied. We found out that above mentioned physical observable depicted a strong dependence on the fourth quark ( $m_{t'}$ ) and the product of quark mixing matrix elements ( $V_{t'b}^* V_{t'd} = r_{db} e^{i\phi_{db}}$ ). We obtained that the study of these readily measurable quantities, specially, for both  $\mu$  case could serve as a good tool to look for physics beyond the SM. More precisely, the results could be used to indirect search to look for fourth generation of quarks.

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